

Diskmath Week 4

Emil Straschil

A dark blue diagonal gradient bar that starts from the bottom left and extends towards the top right, covering the lower half of the slide.

Hello

I am Emil.

I have a **website**:

emils.site (yes thats the URL)

repo: <https://github.com/emil3tr/emil3tr.github.io>

All materials will be uploaded there.

Any Feedback / Questions /
Wishes / ... ?

→ estraschil@student.ethz.ch

→ “Emil” (floxi4) on dinfk-discord

→ Diskmath-questions: ask here
so others can benefit (:

Where are we right now?

Basics

Sets and
Relations

Number Theory

Algebra

Logic

Abstraction

Formulas

Statements

Prop. Logic

Pred. Logic

Proof Patterns

Sets

Set Operations

Relations

Equivalence

Partial Order

Functions

Countability

Division

Primes

Modular Arith.

Diffie-Hellman

Monoids

Groups

Euler Totient

RSA

Rings

Polynomials

Finite Fields

Err. Corr. Codes

Proof Systems

Logic

Calculi

Res. Calculus

Prop. Logic

Pred. Logic



Today

Chapter 3 is one of the easier topics. You can collect lots of points on the exam here!

1. Questions?
2. Kahoot: Set Basics
3. Recap: Relations + Set Proofs
4. Exercises: Set Properties

Questions?

Some Content for Set Theory

discmath.ch as always

my mega summary contains a
good cheatsheet

formulas vs. statements from
session 3

This is the first topic where you can
look at an old exam to get an idea
of what will be asked

Kahoot

Proofs with Sets

Most proofs are of this kind:

1. set-language $\rightarrow$ logic language
2. work with logic
3. logic language $\rightarrow$ set-language

Use the definitions to transform expressions from set-language to logic-language and vice versa

Proving Subsets

Want to show $A \subseteq B$?

→ show that any element in A is also in B

→ take some x in A and show that this x is also in B

Proving $A = B$

Show that both A is subset of B
and B is subset of A

Relations

$A \times B$

Tuple = ordered set

$\{a, b\} = \{b, a\}$ but $(a, b) \neq (b, a)$

$A \times B$ contains all tuples (a, b)
where a is in A and b in B

Relations

Relation from A to B is simply a subset of $A \times B$.

Can write:

- Literally $\{(a,b), (c,d), \dots\}$
- As a Matrix
- As a graph

Relation over $A \times A$ is a directed graph

Relation $p \subseteq A \times A$ is

reflexive

if for all a in A the tuple (a, a) is in the relation.

Graph: every node has an arrow to itself

symmetric

if (a, b) is in the relation then (b, a) is also in the relation.

Graph: if arrow from a to b then also from b to a

antisymmetric

if NOT (a, b) and (b, a) are in the relation for all a, b .

Graph: if arrow from a to b then no arrow from b to a

NOT THE NEGATION OF SYMMETRIC! A relation can be neither symmetric nor antisymmetric

transitive

if (a,b) and (b,c) in the relation then (a,c) must also be in the relation.

Graph: if arrow from a to b and then from b to a then there is also an arrow from a to c .

$a \rightarrow b \rightarrow c$ implies $a \rightarrow c$

Exercises (HS22 4.7)

a) Prove or disprove: For any sets A, B, C and D we have

$$A \subseteq B \text{ and } C \subseteq D \text{ and } B \cap D \neq \emptyset \implies A \cap C \neq \emptyset.$$

b) Prove or disprove: For any sets A, B and C we have

$$A \cup C = B \cup C \text{ and } A \cap C = B \cap C \implies A = B.$$

- a) We disprove the claim with a counterexample. Let $B = D = \{0\}$ and $A = C = \emptyset$. Then, we have

$$\emptyset = A \subseteq B \text{ and } \emptyset = C \subseteq D \text{ and } \{0\} = B \cap D \neq \emptyset.$$

However,

$$A \cap C = \emptyset.$$

b) We prove the claim as follows. Let A , B , and C be arbitrary sets and assume

$$A \cup C = B \cup C \text{ and } A \cap C = B \cap C.$$

In the following, we prove that $A \subseteq B$. Assume that $x \in A$.

$$x \in A$$

$$\implies x \in A \vee x \in C \quad (\text{logic})$$

$$\iff x \in A \cup C \quad (\text{definition of } \cup)$$

$$\iff x \in B \cup C \quad (A \cup C = B \cup C)$$

$$\iff x \in B \vee x \in C \quad (\text{definition of } \cup)$$

$$\iff x \in B \vee (x \in A \wedge x \in C) \quad (\text{logic, assuming } x \in A)$$

$$\iff x \in B \vee x \in A \cap C \quad (\text{definition of } \cap)$$

$$\iff x \in B \vee x \in B \cap C \quad (A \cap C = B \cap C)$$

$$\iff x \in B \vee (x \in B \wedge x \in C) \quad (\text{definition of } \cap)$$

$$\implies x \in B. \quad (\text{logic})$$

We have thus proved that for any sets A , B and C we have

$$A \cup C = B \cup C \text{ and } A \cap C = B \cap C \implies A \subseteq B.$$

Finally, observe that the other direction $B \subseteq A$ can be obtained simply by swapping A and B . This concludes the proof.